

Harjoitus 5

17. syyskuuta 2014 12:00

Muunnoksen määrittämä luontomomenteen sivulta 50:

$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})}$$

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x, y) = K_{xy}$$

Vain nolasta poikkeavat arvot merkitsevät summalauseissa, joten kuhunkin $F(u, v)$:n alkioon tulevat vain ne termit $f(0, 0) = 1$, $f(2, 1) = 2$ ja $f(2, 2) = 1$. Muut termit ovat nollia. Siispä voimme sieventää F :n lausekkeen muotoon

$$F(u, v) = \frac{1}{9} (1 + 2e^{-j2\pi(\frac{2u+1v}{3})} + e^{-j2\pi(\frac{2u+2v}{3})})$$

$$\begin{aligned} & e^{-j2\pi(\frac{0*0}{3} + \frac{0*0}{3})} \\ & + 2e^{-j2\pi(\frac{0*2}{3} + \frac{0*1}{3})} \\ & + e^{-j2\pi(\frac{0*2}{3} + \frac{0*2}{3})} \end{aligned} = 4$$

$$\begin{aligned} & e^{-j2\pi(\frac{1*0}{3} + \frac{0*0}{3})} \\ & + 2e^{-j2\pi(\frac{1*2}{3} + \frac{0*1}{3})} \\ & + e^{-j2\pi(\frac{1*2}{3} + \frac{0*2}{3})} \end{aligned} = 1 + 3e^{-j\frac{4}{3}\pi}$$

$$\begin{aligned} & e^{-j2\pi(\frac{2*0}{3} + \frac{0*0}{3})} \\ & + 2e^{-j2\pi(\frac{2*2}{3} + \frac{0*1}{3})} \\ & + e^{-j2\pi(\frac{2*2}{3} + \frac{0*2}{3})} \end{aligned} = 1 + 3e^{-j\frac{8}{3}\pi}$$

$$\begin{aligned} & e^{-j2\pi(\frac{0*0}{3} + \frac{1*0}{3})} \\ & + 2e^{-j2\pi(\frac{0*2}{3} + \frac{1*1}{3})} \\ & + e^{-j2\pi(\frac{0*2}{3} + \frac{1*2}{3})} \end{aligned} = 1 + 2e^{-j\frac{2}{3}\pi} + e^{-j\frac{4}{3}\pi}$$

$$\begin{aligned} & e^{-j2\pi(\frac{1*0}{3} + \frac{1*0}{3})} \\ & + 2e^{-j2\pi(\frac{1*2}{3} + \frac{1*1}{3})} \\ & + e^{-j2\pi(\frac{1*2}{3} + \frac{1*2}{3})} \end{aligned} = 1 + 2e^{-j2\pi} + e^{-j\frac{8}{3}\pi}$$

$$\begin{aligned} & e^{-j2\pi(\frac{2*0}{3} + \frac{1*0}{3})} \\ & + 2e^{-j2\pi(\frac{2*2}{3} + \frac{1*1}{3})} \\ & + e^{-j2\pi(\frac{2*2}{3} + \frac{1*2}{3})} \end{aligned} = 1 + 2e^{-j2\pi} + e^{-j4\pi}$$

$$\begin{aligned} & e^{-j2\pi(\frac{0*0}{3} + \frac{2*0}{3})} \\ & + 2e^{-j2\pi(\frac{0*2}{3} + \frac{2*1}{3})} \\ & + e^{-j2\pi(\frac{0*2}{3} + \frac{2*2}{3})} \end{aligned} = 1 + 2e^{-j\frac{4}{3}\pi} + e^{-j\frac{8}{3}\pi}$$

$$\begin{aligned} & e^{-j2\pi(\frac{1*0}{3} + \frac{2*0}{3})} \\ & + 2e^{-j2\pi(\frac{1*2}{3} + \frac{2*1}{3})} \\ & + e^{-j2\pi(\frac{1*2}{3} + \frac{2*2}{3})} \end{aligned} = 1 + 2e^{-j\frac{8}{3}\pi} + e^{-j4\pi}$$

$$\begin{aligned} & e^{-j2\pi(\frac{2*0}{3} + \frac{2*0}{3})} \\ & + 2e^{-j2\pi(\frac{2*2}{3} + \frac{2*1}{3})} \\ & + e^{-j2\pi(\frac{2*2}{3} + \frac{2*2}{3})} \end{aligned} = 1 + 2e^{-j4\pi} + e^{-j\frac{16}{3}\pi}$$

>> fft2(K)

ans =

4.0000 + 0.0000i -0.5000 + 2.5981i -0.5000 - 2.5981i
 -0.5000 - 0.8660i 2.5000 - 0.8660i 1.0000 + 1.7321i
 -0.5000 + 0.8660i 1.0000 - 1.7321i 2.5000 + 0.8660i